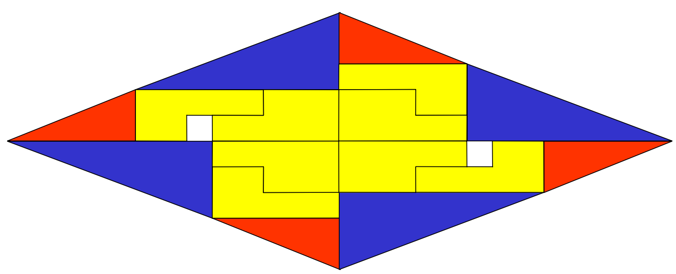
The Air Force Research Laboratory

The Air Force Brain Booster Book

Puzzles, Patterns, and Curios



Compilation by John C. Sparks: AFRL/XPX

The Air Force
Brain Booster Book

Air Force Publication 2006

Dedication

The Air Force
Brain Booster Book

Is dedicated to all Air Force families

O Icarus...

I ride high...
With a whoosh to my back
And no wind to my face,
Folded hands
In quiet rest—
Watching...O Icarus...
The clouds glide by,
Their fields far below
Of gold-illumed snow,
Pale yellow, tranquil moon
To my right—
Evening sky.

And Wright...O Icarus...
Made it so—
Silvered chariot streaking
On tongues of fire leaping—
And I will soon be sleeping
Above your dreams...

100th Anniversary of Powered Flight
1903—2003

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Introduction

The Air Force Brain Booster Book is a collection of fifty-two activities loosely placed into three categories: puzzles, patterns, or curios. The puzzles exercise the use of various logical and problem-solving skills as taught in mathematics and English. Since "pattern" is a basic ingredient that we human seek to find in the world, the patterns and curios within these pages allow both student and teacher to casually explore selected numerical or visual relationships. Some of these relationships could be called amusing, and others quite amazing—but none disappointing. Since traditionally mathematics has been one of the primary tools used to explore patterns, many of the patterns and curios presented are mathematical in nature.

The Air Force Brain Booster Book focuses on problems that can be explored or solved using skills learned in Grades K-12. In the major section "Puzzles, Patterns, and Curios", teachers of these same grades will find plenty of gems that can be used in the classroom both as enrichment and skill-building activities. All problems and activities are identified per the subject abbreviation table found on Page 7 and repeated on Page 41. Where topics and/or problems fit one or more subject categories, subject abbreviations are separated by commas. Everyone will enjoy something from within these pages! For example, children love magic squares, the Kaprekar process, and Lewis Carol's word morphing. Likewise, logic enthusiasts will find several new challenges awaiting them—such as "The Camel and the Bananas" on Page 35.

For those who want more challenging problems involving advanced algebra, calculus, and statistics, the section entitled "Headier Challenges" has some worthy head scratchers, some of which are in the form of actual mathematics tests. Either way, there are certainly enough puzzles to keep both young and old curled up in a chair or bent over a page—eager pencil in hand—for a long, long time.

John C. Sparks: March 2006

Puzzles, Patterns, And Curios

Subject Abbreviations

Subject	**Abbr.**
Logic	*LG*
Arithmetic	*AR*
Algebra	*AL*
Geometry	*GE*
Curios	*CU*
Langage	*LA*
Word Play	*WP*
Calculus	*CL*
Statistics	*ST*

1. The Old Glory Puzzle *LG, GE*

September 11, 2001 is a date that we baby boomers will remember in much the same fashion that our parents remembered December 7, 1941. Our flag is once again enjoying a newfound popularity! Early baby boomers, such as myself, were born under a forty-eight star flag. This flag was arranged in six rows of eight stars each. Hawaii joined the Union in 1959, leading to a forty-nine star flag—seven rows of seven stars each. Alaska joined the Union one year later, leading to the present fifty star flag arranged in nine slightly nested rows alternating seven, six, seven, six, seven, six, seven, six, and seven stars.

Suppose new states are added to the Union during the current century. Possibilities might include Puerto Rico, Guam, and the District of Columbia. *Here is your challenge:*

Arrange three rectangular fields to accommodate fifty-one, fifty-two, and fifty-three stars. Use the dual constraint that there shall be no more than nine rows and no more than eight stars per row, a historical precedent. Having problems? Step away from the rectangular pattern—literally, out-of-the-box thinking—and go to a circular pattern, utilized at least once in our nation's history.

Finally, for those of you who need even more of a challenge, keep on adding the states and stars all the way to seventy-two—nine times eight.

2. Twin Towers Numerology *CU, AR*

The following number curios were sent to me shortly after the World Trade Center bombing. We humans are always seeking meaning and significance in those tragic or happy events that affect our lives. The establishing of numerical patterns is one way (and a very ancient one) that people use to explore meaning.

- The date of the attack: 9/11 = 9+1+1 = 11
- September 11, 254th day of the year: 2+5+4 = 11
- After September 11, 111 days are left in the year
- 119 is the area code for Iran/Iraq: 1+1+9 = 11
- The Twin Towers looked like the number 11
- The first plane to hit the towers was Flight 11
- The 11th state added to the Union was New York
- New York City has 11 letters
- Afghanistan has 11 letters
- “The Pentagon” has 11 letters
- Ramzi Yousef has 11 letters (convicted in the World Trade Center bombing of 1993)
- Flight 11 had 92 people on board: 9+2 = 11
- Flight 77 had 65 on board: 6+5 = 11

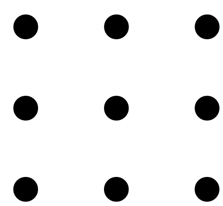
A Question to Ponder: Is there a significant event in your life where you have used numerical patterns (perhaps dates) to help establish meaning? Martin Gardner (of Scientific American fame) has given the name jiggery-pokery to such quests for numerological patterns. Jiggery-pokery or not, we will continue as rational beings to seek and establish patterns of all sorts.

II

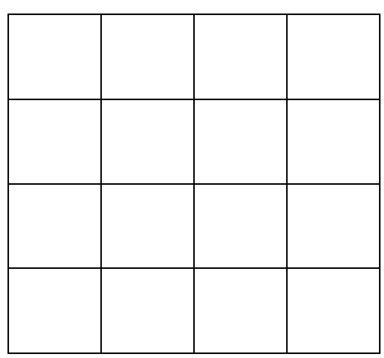
3. Two Squares and Two Challenges *LG, AR*

The following two puzzles have been used as icebreakers for years in various group settings. Both are simple yet profound and illustrate the use of out-of-the-box or lateral thinking.

A) Try to connect all nine dots using just four straight-line segments and one continuous pen stroke.



B) Count the total number of squares contained in the big square below.



4. Ten Commandments of Algebra *AL*

Algebra can be thought of as a language, universal in scope! Many people are frustrated when learning this language because they fail to follow a few basic study rules. Here are ten such rules written in yesterday's English.

1. Thou shall read thy problem.
2. Whatsoever thou shall do to one side of the equation, do thou also to the other side.
3. Thou shall draw a picture when thou tackles a word problem in order to actively engage both sides of thy brain.
4. Thou shall ignore the teachings of false prophets to do complicated work in thy head.
5. Thou must use thy "Common Sense", or else thou wilt have flagpoles 9000 feet in height, yea…even fathers younger than sons.
6. When thou does not know, thou shall look it up; and if thy search is fruitless, thou shall ask the teacher.
7. Thou shall master each step before putting down in haste thy heavy foot on the next.
8. Thy correct answer does not always prove that thou has understood or correctly worked the problem.
9. The shall first see that thou has copied thy problem correctly before bearing false witness that the book is a father of lies.
10. Thou shall look back even to thy youth and remember thy arithmetic.

5. Numerology of 666 *AR, CU*

The number 666 has been in Western thought for about twenty centuries. In Roman times, 666 would have been easily written in Roman times as DCLXVI, a simple descending sequence of the first six Roman numerals. Today, writing 654321 would serve the same purpose. The number 666 has many fascinating numerical properties, eight of which are listed below.

1. $666 = 6 + 6 + 6 + 6^3 + 6^3 + 6^3$
2. $666 = 1^6 - 2^6 + 3^6$
3. $666 = 2^2 + 3^2 + 5^2 + 7^2 + 11^2 + 13^2 + 17^2$: The sum of the squares of the first seven prime numbers
4. $666 = 313 + 353$: The sum of two consecutive prime numbers that read the same forward and backward).
5. $666 = 2x3x3x37$ and $6 + 6 + 6 = 2 + 3 + 3 + 3 + 7$. 666 is called a Smith number since the sum of its digits is equal to the sum of the digits of its prime factors.
6. $666^2 = 443556$ and $666^3 = 295408296$. Furthermore, $(4^2 + 4^2 + 3^2 + 5^2 + 5^2 + 6^2) +$ $(2 + 9 + 5 + 4 + 0 + 8 + 2 + 9 + 6) = 666$
7. 666 is made from the ascending sequence 123456789 by insertion of one or more plus signs in two different ways: $1 + 2 + 3 + 4 + 567 + 89 = 666$ $123 + 456 + 78 + 9 = 666$
8. Likewise, 666 is made from the descending sequence 987654321 by $666 = 9 + 87 + 6 + 543 + 21$.

Challenge: Are there other ways that you can make the number 666 from the sequences in 7) and 8) using only plus signs? Using both plus and minus signs?

6. Can You Pass a Simple F Test? *WP, AR*

> We have the opportunity of looking at farming and the future of farmers. How do we handle the training of farm hands? Too, how to encourage the use of farm fundamentals? How do we find right ways of improving future farm animals? How can we find mechanisms that allow low-cost transfer of the farm from one farm family to another farm family? This is the basis of effective farm management and the future of financially sound farms.

Challenge: Read the message in the shaded frame above, and count the number of times the letter f appears in the message.

7. Two is Equal to One! *AL*

What is wrong with the following demonstration that algebraically shows—without a doubt—that two is equal to one? *Hint*: Think back to the cardinal no-no of algebra!

Demonstration that Two Equals One

1. Set $x = y$
2. Multiply both sides by x: $x^2 = yx$
3. Subtract y^2 from both sides: $x^2 - y^2 = xy - y^2$
4. Factor both sides: $(x - y)(x + y) = y(x - y)$
5. Divide both sides by $x - y$: $x + y = y$
6. But $x = y$ by statement 1. Substituting, $2y = y$.
7. Dividing both sides by y, we have $2 = 1$. $\therefore$

8. Crossing Problems Old and New *LG*

Logic problems where several animals, objects, and/or people must cross over a river under a set of constraints have entertained and baffled puzzle solvers for many centuries. Below are two such problems. The Wolf, Goat, and Cabbage problem is at least 1000 years old, and urban legend has it that the "U2" Concert problem was a question on a Microsoft employment exam. Enjoy the two challenges!

A) Wolf, Goat, and Cabbage

A farmer and his goat, wolf, and cabbage come to a river that they wish to cross. There is a boat, but it only has room for two, and the farmer is the only one who can row. However, if the farmer leaves the shore in order to row, the goat will eat the cabbage, and the wolf will eat the goat. Devise a minimum number of crossings so that all concerned make it across the river alive and in one piece.

B) The "U2" Concert

"U2", the four-man Irish rock band, has a concert that starts in 17 minutes, and they all must cross a bridge in order to get there. All four men begin on the same side of the bridge. Your job is to devise a plan to help the group get to the other side on time. There are several constraints that complicate the crossing process: It is night and a flashlight must be used, but there is only one flashlight available. Any party that crosses—only 1 or 2 people allowed on the bridge at any given time—must have the flashlight with them. The flashlight must be walked back and forth; it cannot be thrown, teleported, etc. Each band member walks at a different speed, and a pair walking together must cross the bridge using the slower man's speed. Here are the four crossing times: Bono takes 1 minute to cross; Edge, 2 minutes to cross; Adam, 5 minutes to cross; and Larry, 10 minutes to cross. Devise the plan!

9. President Garfield and Pythagoras *GE, AL*

The Pythagorean Theorem was known in antiquity at least 1000 years before Pythagoras (circa 600 BC), the Greek mathematician who developed the first known proof. Proving the Pythagorean Theorem—especially constructing new proofs—has been a source of intellectual entertainment for many centuries. President Garfield constructed an original proof based on trapezoid properties while still a Congressman. Today, there are over 300 known proofs of the Pythagorean Theorem. *Challenge*: Using the two figures below as your guide, can you reconstruct the associated unique proofs of the Pythagorean Theorem? Figure 1 was the one used by President Garfield and Figure 2 has its origins in ancient China.

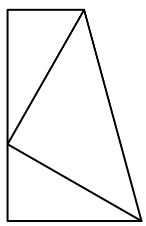


Figure 1

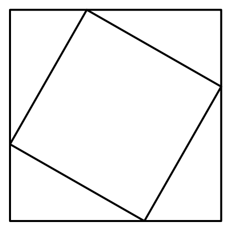
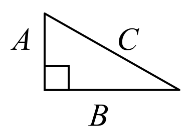


Figure 2



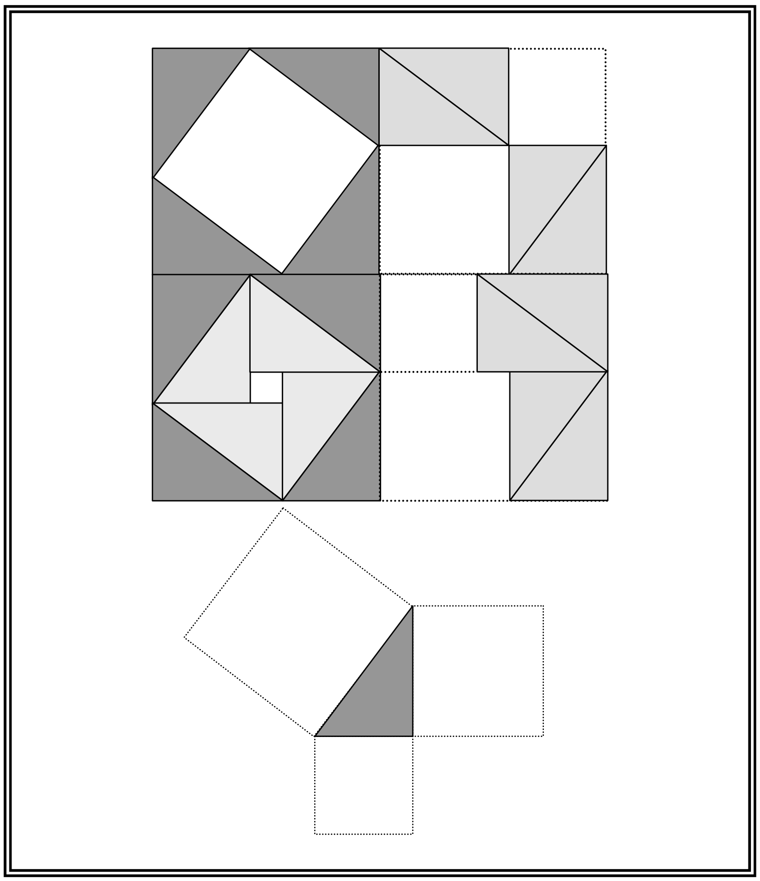
Statement of the Pythagorean Theorem

Let a right triangle have sides of lengths $A, B,$ and C where C is the hypotenuse. Then the following relationship holds:

$$C^2 = A^2 + B^2.$$

10. Pythagorean Mosaic *LG, GE*

The following mosaic illustrates the truth of the Pythagorean Theorem without resorting to the use of algebra. Thus, it can be used to demonstrate the Pythagorean Theorem to younger children or to those who do not have a working knowledge of algebra.



Challenge: Study the mosaic and deduce the truth of the Pythagorean Theorem through simple “take-away” logic alone.

11. A Potpourri of Powers *AR, CU*

After seeing the Pythagorean relationship between three squares, one might ask what other neat relationships exist amongst numbers, multiples, digits, and powers. Below is a connoisseur's sampling. Enjoy!

1. $6^3 = 3^3 + 4^3 + 5^3$
2. $49 = 47 + 2$ and $94 = 47x2$
3. $371 = 3^3 + 7^3 + 1^3$ and $407 = 4^3 + 0^3 + 7^3$
4. $135 = 1^1 + 3^2 + 5^3$ and $175 = 1^1 + 7^2 + 5^3$
5. $169 = 13^2$ and $961 = 31^2$
6. $244 = 1^3 + 3^3 + 6^3$ and $136 = 2^3 + 4^3 + 4^3$
7. $499 = 497 + 2$ and $994 = 497x2$
8. $504 = 12x42 = 21x24$
9. $1634 = 1^4 + 6^4 + 3^4 + 4^4$ and $3435 = 3^3 + 4^4 + 3^3 + 5^5$
10. $2025 = 45^2$ and $20 + 25 = 45$
11. $4913 = 17^3$ and $4 + 9 + 1 + 3 = 17$
12. 9240 has 64 divisors! Can you find them all?
13. $54{,}748 = 5^5 + 4^5 + 7^5 + 4^5 + 8^5$
14. $321489 = 567^2$ Not counting the exponent 2, this equality uses each of the nine digits just once. The only other number that does this is 854.

Challenge: If you don't believe one of the statements below, then check it out.

12. The 3X3 Magic Square *AR, CU*

Kids love magic squares, and magic squares are a great way to encourage a child to practice addition skills. Start them out on the 3X3 magic square shown below, which uses the numbers 1 through 9.

2	7	6
9	5	1
4	3	8

Explain to them that if we add all the numbers in any one row or column, the sum is always 15 (called the magic constant). The same is true for the numbers on the two diagonals. Two questions I like ask are, "Can you show this is true?" and "How many different ways can we make a sum of 15?" An interested fact is that we have records of 3X3 magic squares dating back to 1000 BC.

A cool challenge for the young math student: Add 10 to each number in the magic square above. Is the new square also a magic square? If so, what is the magic constant?

13. The 3X3 Anti-Magic Square *AR, CU*

Per the previous entry, the 3X3 magic square has the property that the three numbers in each of the three rows, three columns, and two diagonals (eight sums altogether) add up to 15. The 3x3 Anti-Magic Square has *no two sums alike*.

Challenge: Using the digits one through nine, construct an opposing 3X3 anti-magic square where no two sums are alike.

2	7	6
9	5	1
4	3	8
?	?	?
?	?	?
?	?	?

14. A Perfect 4X4 Magic Square *AR, CU*

Once students have explored the 3X3 magic square, introduce them to the 4 by 4 magic square shown below, which uses the numbers 1 through 16 and has a magic constant of 34.

1	15	6	12
8	10	3	13
11	5	16	2
14	4	9	7

In this 4X4 (as in the 3X3) all rows, columns, and diagonals sum to the magic constant. But…that is only the beginning of the story. The four corners of the 4X4 square also sum to the number 34. Now, continue by adding the four corner numbers of *any sub-square of any size contained anywhere in the big square*. These four corner numbers also sum to the magic constant 34. A 4x4 magic square that has this amazing four-corner number-summing property is further classified as a 4X4 Perfect Magic Square.

Challenge: In how many different ways can the number 34 be made? This ought to keep those kids—and their teacher—busy for a while!

Super Challenge: Can you find any other geometric patterns of four numbers within this square which also sum to 34. Several such patterns are shown below. There are more!

	X		O
O		X	
	O		X
X		O	

X		X	
	O		O
X		X	
	O		O

	X	O	
X			O
O			X
	O	X	

X		O	
	O		X
O		X	
	X		O

15. Two 5 by 5 Magic Squares *AR, CU*

The first magic square shown below is a 5x5 magic square where the magic constant is 65. The second magic square is called a 5x5 nested magic square in that the shaded 3x3 square within the 5x5 square is also a magic square.

1	15	8	24	17
23	7	16	5	14
20	4	13	22	6
12	21	10	19	3
9	18	2	11	25

Challenge: For the 5x5 magic square above, in how many different ways can the magic sum be made using five numbers?

1	18	21	22	3
2	10	17	12	24
18	15	13	11	8
21	14	9	16	5
23	7	6	4	25

Challenge: Find the magic sum for the shaded 3x3 nested square.

16. Ben Franklin's 8X8 Magic Square *AR, CU*

Benjamin Franklin loved magic squares. In 1769, he invented the 8X8 magic square shown below using the counting numbers 1 through 64. Now Ben's square has a little problem: if you sum the numbers for each row, column, and diagonal (18 different ways altogether), the totals miss the mark in two cases. I guess he had a hard time fitting it all in—perhaps leading to some very long and sleepless nights!

Challenge: Can you discover the two inconsistencies in old Ben's square? *Hint*: you may want to review your addition skills before you start.

52	61	04	13	20	29	36	45
14	03	62	51	46	35	30	19
53	60	05	12	21	28	37	44
11	06	59	54	43	38	27	22
55	58	07	10	23	26	39	42
09	08	57	56	41	40	25	24
50	63	02	15	18	31	34	47
16	01	64	49	48	33	32	17

Super Challenge: Is there such a thing as a 2 by 2 magic square using the counting numbers 1, 2, 3, and 4? Why or why not?

17. Kaprekar Teaches Subtraction (Part 1) *AR, CU*

Shri Dattathreya Ramachanda Kaprekar was an Indian mathematician who discovered a fascinating "number-crunching" process in the late 1940s that involved the use of three and four digit numbers. Today, Kaprekar's Process is a wonderful tool (or game) that can aid in the building of subtraction skills.

Take any three-digit number whose digits are not all the same (222 is not OK, but 221 is OK). Rearrange the digits twice in order to make the largest and smallest numbers possible. Subtract the smaller number from the larger. Take the result and repeat the process. Let's see what happens for three different three-digit numbers!

517	**263**	**949**
751-157=594	632-236=396	994-499=545
954-459=495	963-369=594	554-455=099
954-459=**495**	954-459=**495**	990-099=891
		981-189=792
		972-279=693
		963-369=594
		954-459=**495**

Notice that the cycle ends (or stalls) at the number **495** (called the Kaprekar constant) each time we run the process. Point: Remember to write two-digit results using three digits (i.e. 99 becomes 099 in the **949** cycle).

Challenge: Try it, and have your students try it. Count the cycles to reach **495.** You may want have a subtraction race.

18. Kaprekar Teaches Subtraction (Part 2) *AR, CU*

Kaprekar's process also works for four-digit numbers where the Kaprekar constant is **6174**. Let's run Kaprekar's process on the number **1947** (my birth year) which suggests several great subtraction activities for your classroom.

1947

9741-1479=8262

8622-2268=6354

6543-3456=3087

8730-0378=8352

8532-2358=6174

7641-1467=**6174**

Potential activities

- *Run the process for your birth year*
- *Count the cycles to reach* **6174**
- *Which student has the most cycles?*
- *Which student has the least cycles*?

Challenge: There is also a Kaprekar constant for two-digit numbers. Can you find it?

19. Perfect Numbers Big and Small *AR, CU*

Do you have little angels your classroom? Probably not! But, there are numbers which are called perfect. A perfect number is simply a number that is equal to the sum of all of its proper divisors (a proper divisor of a number is any divisor smaller than the number itself). The first perfect number is **6** since 6=1+2+3 (all divisors smaller than 6). To show **28** is perfect, just note that 28=1+2+4+7+14. Perfect numbers are quite rare—such as perfect children—and grow rapidly in size. Below is a list of the first seven perfect numbers.

6: known to the ancient Greeks

28: known to the ancient Greeks

496: known to the ancient Greeks

8128: known to the ancient Greeks

33550336: recorded in medieval manuscript

8589869056: Cataldi discovered in 1588

137438691328: Also discovered by Cataldi in 1588

Challenge: Can you or your class show that **496** is a perfect number? Do you dare examine **8128**!

20. Are You Abundant or Deficient? *AR, CU*

Any number, such as your birth year, has proper divisors. If all proper divisors of a number sum to more than the number itself, the original number is called abundant. Deficient numbers are where the opposite is true (sum to less than the number itself). All prime numbers are deficient.

Challenge: Have each member of your class take their birth year and see if it is abundant or deficient. Who has the most abundant number (exceeds the birth year by the greatest amount)? Abundant or not, I am feeling pretty deficient when I have the flu! How about you?

21. Friendly Pairs *AR, CU*

A pair of numbers is called friendly if each number in the pair is the sum of the proper divisors for the other number. The first and smallest friendly pair is **220** and **284**, discovered by the Greeks. Take a look:

220=1+2+4+71+142 (all of the proper divisors of 284)

284=1+2+4+5+10+11+20+22+44+55+110 (all of the proper divisors of 220)

Three other pairs of friendly numbers are shown below. Today, over 1000 pairs of friendly numbers are known.

1184 & **1210**: discovered by Paganini in 1866 at age 16

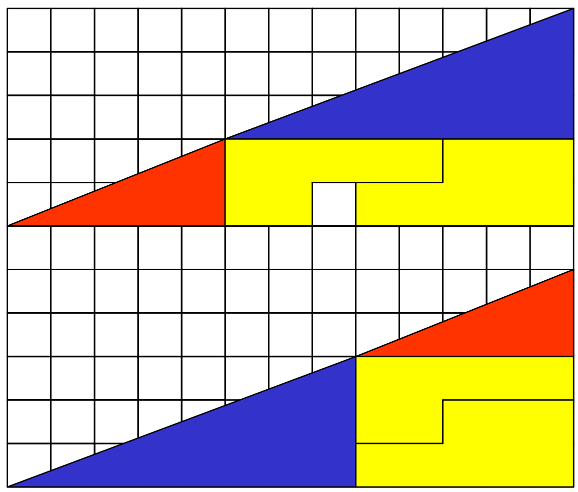
17,163 & **18,416**: discovered by Fermat in 1636

9,363,584 & **9,437,056**: discovered by Descartes in 1638

Challenge: Can you show that **1184** and **1210** are friendly?

22. Alfred Curry's Missing Area Paradox *GE*

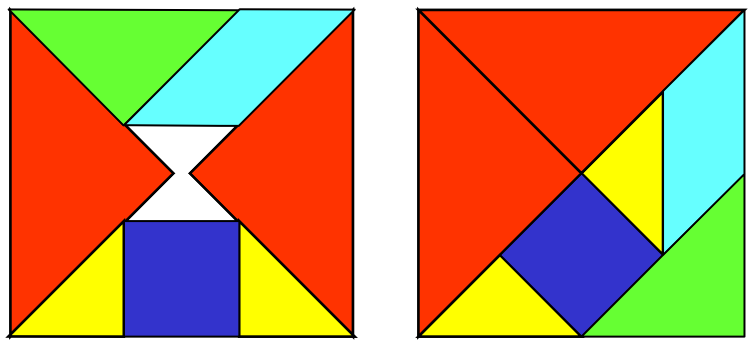
Alfred Curry was an amateur magician and mathematician who lived in New York City. Circa 1950, he invented the geometric paradox shown below. Each of the two triangular arrangements shown below uses the same four pieces. But alas! One of the areas has a little square missing.



Challenge: How did the square disappear?

23. Tangram Missing Hole Paradox *GE*

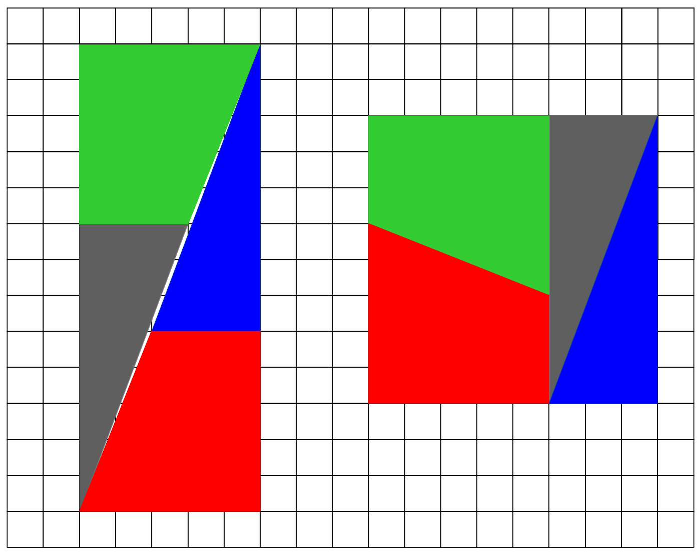
Identical Tangram sets consisting of the seven traditional pieces are used to construct both figures.



Challenge: How did the irregular-shaped hole disappear?

24. Square and Rectangle Paradox *GE*

Two identical geometric sets, each consisting of four planar pieces, are used to construct a square and rectangle with differing areas.



Challenge: Resolve the paradox

25. Word Morphing with Lewis Carroll *WP, LA*

Lewis Carroll—mathematician, teacher, and author of Alice in Wonderland—invented a marvelous word game in the 1870s that he called “Doublets”. Nowadays, I’ll call it word morphing. Here is how it goes: Take two words having the same number of letters, say **cat** and **dog**. Can you transform (morph) the **cat** into a **dog** by changing only one letter at a time where each intermediate form is a bona-fide word in the English language? **Cat** and **dog** are easy. Consider the sequence--**cat**, **cot**, **dot**, and **dog**—which solves the problem quite nicely. Again, every word in the sequence (Carroll called this a chain) must be an English word, and the player can only change one letter at a time. Also, the original rules prohibit switching letters within a word. Here is another example; to turn **warm** into **cold**, construct the sequence: **warm**, **ward**, **card**, **cord**, and **cold**. One can have fun anywhere and almost anytime with Lewis Carroll’s wonderful little word game!

Challenge: Back in Carroll’s day, nobody could take the **horse** to the **field**. I am now told that English words are available that can make this chain happen—your move!

26. The Three Bears *WP, LA*

Which of the following sentences, if any, bears errors?

1) No bear bare should bare a burden.
2) No bear bare should bear a burden.
3) No bare bear should bear a burden.

27. Prime Time *AR, AL*

A prime number is an integer (or counting number) that has no proper divisors other than one. Hence, by definition, all prime numbers are deficient. Even so, mathematicians still love prime numbers and have been fascinated by their properties for many years. Below is a table showing the prime numbers less than 100.

2	3	5	7	11
13	17	19	23	29
31	37	41	43	47
53	59	61	67	71
73	79	83	89	97

The mathematician Leonhard Euler (1707-1783) discovered the following simple formula that generates prime numbers P for all counting numbers x starting with 0 and continuing through 39:

$$P = x^2 + x + 41.$$

For example, when $x = 5$, we have that $P = 5^2 + 5 + 41 = 71$, a prime number!

The above formula can be used in at least two skill-building activities suitable for beginning algebra students.

Challenge: Use Euler's formula above to generate all 39 prime numbers. Once generated, check each number to insure that it is indeed prime.

Super Challenge: Set P equal to each prime number in the table above and solve for the variable x that makes it so!

28. Here Lies Old Diophantus *AL*

The Greek Mathematician Diophantus of Alexander (born about 200 AD) is considered by many historians to be the father of algebra. He wrote a book called Arithmetica, the earliest written record containing variables, algebraic equations, and solutions. There is an epithet for Diophantus (published about 500 AD) describing his life in terms of a riddle: “This tomb holds Diophantus. Ah, how great a marvel! The tomb tells scientifically the measure of his life. Zeus granted him to be a boy for one-sixth of his life, and adding a twelfth part to this, Zeus clothed his cheeks with down. He lit him the light of wedlock after a seventh part, and five years after his marriage Zeus gave him a son. Alas, late-born wretched child! After obtaining the measure of half his father’s life, chill Fate took him. After, consoling his grief by the study of numbers for four years, Diophantus ended his life.”

Challenge: From this riddle, can you determine how old Diophantus was when he died?

29. Young Gauss Stuns His Teacher *AR, EA*

Carl Gauss (1777-1855) is considered by many to be one of the greatest mathematicians of all time. Legend has it that he entered school at the age of 5 and stunned his teacher who gave him a tedious problem to solve, thinking it would take the lad an hour or more. Here is the problem: add the counting numbers 1 through 100. The answer is 5050, and the young five-year-old Gauss had determined it within one minute!

Challenge: How did young Gauss solve the problem so quickly? Can you extend his technique to add the counting numbers 1 though 2006?

30. One Dollar Please *LG*

The following logic puzzle is very old. It always seems to challenge each new generation of thinkers as the story line gets updated to fit changing times. Three men stayed for one night in a motel, all three sharing the same room. They checked out the next morning, the bill for the night coming to $25.00. Each man gave the motel clerk a $10.00 bill and told him to keep $2.00 of the change as a tip. The clerk gave $1.00 in change back to each of the three men. A quick reckoning has the night costing $27.00 plus a $2.00 tip. Where did the other dollar go?

31. The Four Fours Puzzle *AR, AL*

Challenge: Create all the counting numbers 0 through 100 using mathematical equalities having exactly four 4s and no other numerals on the left hand side.

Two examples are

$$4x4x4 - 4 = 60$$

&

$$4 \div 4 + 4! + \sqrt{4} = 27\,.$$

32. Finding Your Palindrome *AR, CU*

A palindrome is a number that reads the same forwards and backwards. 1374731 is a palindrome and so is 1551. Take any number and reverse its digits and add the new number to the original number. Repeat this process. Eventually, this reverse-and-add process has a good chance of producing a palindrome. Out of all counting numbers less than 100,000, only 5996 numbers fail to produce palindromes via this method. 196 is the smallest. Other numbers may require quite a few steps in order to finally reach a palindrome (e.g. 187 takes 23 steps).

Our two sons were born in 1973 and 1980. Let's use the digit reversal process to chase down their palindromes. Fortunately, both of our sons have one, and both birth years take five steps to produce a palindrome.

1973	1980
3791	0891
5764 *step 1*	2871 step 1
4675	1728
10439 *step 2*	4653 step 2
93401	3564
103840 *step 3*	8217 step 3
048301	7128
152141 *step 4*	15345 step 4
141251	54351
293392 *palindrome*	69696 *palindrome*

Challenge: Does your birth year have a palindrome associated with it? Can you find it? How about other members of your family?

33. My Problem with Ice Cream *AR, AL*

My problem with ice cream is that I love it, and I always have! The problem below is for all ice-cream lovers. And, if you are a true ice-cream lover, I can imagine you saying, “Not a problem!”

Challenge: The local ice-cream parlor sells monster once-in-a-lifetime sundaes for those very special occasions. A customer is allowed to pick from three flavors: chewy double chocolate crunch ($1.00 per scoop), multi-berry ambrosia ($1.60 per scoop), and Aegean vanilla ($.80 per scoop). A monster sundae costs $20.00, $16.00 for 15 scoops of ice cream and an additional $4.00 for an assortment of delectable toppings. In how many different ways can I order my monthly treat? What are they? *Note:* the parlor will not serve partial scoops,

34. The Camel and the Bananas *LG, AL*

A camel used to transport bananas must travel 1000 miles across a desert to reach customers living in an exotic city. At any given time, the camel can carry up to 1000 bananas and must eat one banana for every mile it walks.

Challenge: Assuming an initial stock of 3000 bananas, what is the maximum number of bananas that the camel can transport across the desert and into the eager hands of waiting customers who live in the exotic city?

35. Magic or Algebra *AR, AL, CU*

The following little bit of numerical mystery is great fun for those in elementary school. Those a little older may want to ponder whether this little bit of jiggery-pokery is really magic or has its secrets in algebra…

1. Pick a number from 1 to 9
2. Multiply the number by two
3. Add 5
4. Multiply the result by 50
5. Answer the following question yes or no: have you had your birthday this year?
6. Add the magic number as given from the table below!
7. Subtract the four-digit year that you were born

You should end up with a three-digit number. The first digit should be the number that you originally picked and the last two digits should be your age! Wow!

For the year	**If no, add**	**If yes, add**
2006	1755	1756
2007	1756	1757
2008	1757	1758
2009	1758	1759
2010	1759	1760

There are Three Levels of Challenges:

- Elementary school: make it work and impress all of your friends as you perform this trick!
- Middle school: figure out why it works!
- High school: make up your own piece of numerical magic following this general pattern and impress your less-mathematical friends!

36. Word Squares *WP, LA*

Word squares, which were very popular throughout the 1800s, are the language equivalent of magic squares and the forerunners to the modern crossword puzzle. Below are five word squares of various sizes.

2x2
ON
NO

3x3	**4x4**	**5x5**	**6x6**
BAG	LANE	STUNG	CIRCLE
APE	AREA	TENOR	ICARUS
GET	NEAR	UNTIE	RAREST
	EARS	NOISE	CREATE
		GREET	LUSTRE
			ESTEEM

As shown, each square is composed of words of equal length that read in exactly the same way both horizontally and vertically. Diagonals do not have to be words. The 6x6 above is famous because when it first appeared in 1859, it claimed—tongue in cheek—to have solved the problem of "squaring the circle" (see note below). 7x7, 8x8, and 9x9 word squares are in existence today—but no 10x10!

Challenge: Try to construct a word square consisting of words unique to your family, town, favorite sports team, etc. in such a way that all the words support the same general idea. *Super Challenge*: Construct a 10x10 word square and become famous! Gain entry into Ripley's Believe it or Not!

Note: The problem of squaring the circle is an ancient Greek geometric problem that requires the solver to construct a square whose area is equal to that of a given circle using only a compass and straight edge. This problem has since been proven unsolvable under the two classical constraints—compass and straightedge—as given.

37. Narcissistic Numbers *AR, CU*

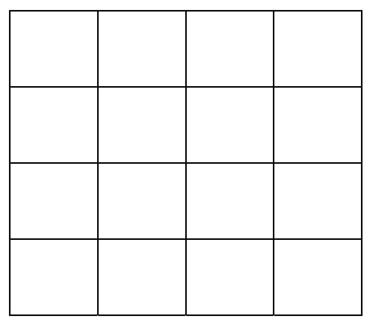
According to Greek mythology, Narcissus fell in love with his own image while looking into a pool of water. He subsequently turned into a flower that "bears" his name. Narcissistic numbers are numbers whose own digits can be used to recreate themselves via established rules of arithmetic. A lovely sampling is below—all waiting to be verified by the willing student! Also included are three examples of "narcissistic pairs".

1. $3435 = 3^3 + 4^4 + 3^3 + 5^5$
2. $127 = -1 + 2^7$
3. $598 = 5^1 + 9^2 + 8^3$
4. $3125 = (3^1 + 2)^5$
5. $1676 = 1^1 + 6^2 + 7^3 + 6^4 = 1^5 + 6^4 + 7^3 + 6^2$
6. $759375 = (7 - 5 + 9 - 3 + 7)^5$
7. $2592 = 2^5 \cdot 9^2$
8. $1233 = 12^2 + 33^2$
9. $990100 = 990^2 + 100^2$
10. $94122353 = 9412^2 + 2353^2$
11. $2646798 = 2^1 + 6^2 + 4^3 + 6^4 + 7^5 + 9^6 + 8^7$
12. $2427 = 2^1 + 4^2 + 2^3 + 7^4$
13. $24739 = 2^4 \cdot 7! \cdot 3^9$
14. $3869 = 62^2 + 05^2$ & $6205 = 38^2 + 69^2$ *pair*
15. $5965 = 77^2 + 06^2$ & $7706 = 59^2 + 65^2$ *pair*
16. $244 = 1^3 + 3^3 + 6^3$ & $136 = 2^3 + 4^3 + 4^3$ *pair*
17. $343 = (3 + 4)^3$
18. $221859 = 22^3 + 18^3 + 59^3$
19. $416768 = 768^2 - 416^2$
20. $3468 = 68^2 - 34^2$

38. Coloring the Grid *LG, GE*

We keep coming back to squares in this little book. This time, the object is to color the 4 by 4 grid shown below where 4 of the little squares are to be blue, 3 are to be green, 3 are to be white, 3 are to be yellow, and 3 are to be red. Oh yes, there is one additional requirement.

Challenge: Color the grid so that no color appears more than once in any horizontal, vertical, or diagonal line.



39. The 100 Puzzle *LG, AR*

The 100 Puzzle is a very old favorite which can be used in the middle grades as an arithmetic enrichment exercise. Here is how it goes. First, write the digits one through nine in natural order. Now, without moving any of the nine digits, insert arithmetic signs and/or parenthesis so that the digits total to 100. Dudeney, one of the greatest puzzles creators of all times, claimed that

$$1+2+3+4+5+6+7+(8x9)=100$$

was the most common solution. He came up with many solutions during his lifetime including this favorite:

$$123-45-67+89=100.$$

Dudeney liked this particular solution because it minimized the number of arithmetic signs.

Yet another solution is $12+3-4+5+67+8+9=100$.

Four challenges:

1) Conduct a classroom contest to see who can come up with the most solutions.
2) Conduct a classroom contest to see who can come up with a solution having a minimum number of signs.
3) Reverse the digits one through nine (writing them in decreasing order) and play The Reverse 100 Puzzle.

   One solution is $98+7-6+5-4+3-2-1=100$.

4) Write the nine digits in random order and play 1) and 2) above.

Headier Challenges

Subject Abbreviations

Subject	**Abbr.**
Logic	*LG*
Arithmetic	*AR*
Algebra	*AL*
Geometry	*GE*
Curios	*CU*
Langage	*LA*
Word Play	*WP*
Calculus	*CL*
Statistics	*ST*

40. 1885 Arithmetic Exam *AR, AL*

If you wanted to *enter* Jersey City High School back in 1885, you first had to pass an entrance exam covering five basic academic disciplines: arithmetic, geography, United States history, English grammar, and algebra. The ten questions below comprise the 1885 arithmetic exam.

1. If a 60-days note of $840.00 is discounted at 4.5% by a bank, what are the proceeds?
2. The interest on $50.00 from 1 March to 1 July is $2.50. What is the annual simple interest rate?
3. The mason work on a building can be finished by 16 men in 24 hours, working 10 hours a day. How long will it take 22 men working 8 hours a day?
4. By selling goods at 12.5% profit, a man clears $800.00. How much did they cost? For how much were they sold?
5. What is the cost of 83 pounds of sugar at $98.50 a ton?
6. A merchant sold some goods at a 5% discount for $18,775.00 and still made a 10% profit. What did the merchant pay for the goods?
7. Find the sum of $\sqrt{16.7281}$ and $\sqrt{.721\frac{1}{4}}$
8. Find $(.37-.095)\div(.00025)$ .Express the result in words.
9. A requires 10 days and B 15 days to paint a house. How long will it take A and B together to paint the house?
10. A merchant offered some goods for $1170.90 cash, or $1206 payable in 30 days. Find the simple interest rate.

41. 1885 Algebra Exam

AR, AL

The ten questions below comprise the algebra portion from the same 1885 Jersey City High School entrance exam.

1. Define algebra, algebraic expression, and polynomial.
2. Simplify the following expression: $1-(1-a)+(1-a+a^2)-(1-a+a^2-a^3)$.
3. Find the product of the two expressions $3+4x+5x^2-6x^3$ and $4-5x-6x^2$.
4. Write a homogeneous quadrinomial of the third degree.
5. Express the cube root of $10ax$ in two ways.
6. Find the prime factors of a) x^4-b^4 and b) x^3-1.
7. Find both the sum and difference of the two expressions
   $3x-4ay+7cd-4xy+16$ &
   $10ay-3x-8xy+7cd-13$.
8. Divide the expression $6a^4+4xa^3-9(ax)^2-3ax^3$ by the expression $2a^2+2ax-x^2$ and check.
9. Find the Greatest Common Divisor (G.C.D.) for the two expressions $6a^2+11ax+3x^2$ and $6a^2+7ax-3x^2$.
10. Divide $\dfrac{x^2-2xy+y^2}{ab}$ by $\dfrac{x-y}{bc}$ and give the answer in its lowest terms.

42. 1947 Algebra Exam *AR, AL*

Below is the algebra portion of a Canadian high school exit exam from the year 1947. A score of 80% was required to pass. How do you score in this century?

1. Prove: $\log_a N^p = p \log_a N$.
2. Plot the graphs of $y = 3x^2 - x^3$ and $y = 3x + 7$ on the same set of axis for the interval $-1 \le x \le 4$. Prove that $y = x^3 - 3x^2 + 3x + 7$ has one real root and find it.
3. If $\frac{x}{y}$ varies as $(x + y)$ and $\frac{y}{x}$ varies as $x^2 - xy + y^2$, show that $x^3 + y^3$ is a constant.
4. Prove: $a + (a + d) + (a + 2d) + ... = \frac{n}{2}\left(2a + [9n - 1]d\right)$.
5. If $P_n^5 = 90 P_{n-2}^3$, find the value of n.
6. How many even numbers of four digits can be formed with the numerals 2, 3, 4, 5, 6, if no numeral is used more than once in each number?
7. If m and n are the roots of the quadratic equation $ax^2 + bx + c = 0$, prove that $m + n = -\frac{b}{a}, mn = \frac{c}{a}$.
8. One root of the equation $x^2 - (3a + 2)x + 12 = 3$ is three times the other. Find the value of a.
9. Expand $\frac{1}{(1 - 3x)^2}$ to 4 terms in ascending powers of x.
10. Show that when higher powers of x can be neglected, $\frac{\sqrt{1 + x} + \sqrt[3]{(1 - x)^2}}{1 + x + \sqrt{1 + x}}$ is approximately $1 - \frac{5}{6}x$.

43. 2006 Expert-Level Algebra Exam *AR, AL*

1. Solve a) for w. Solve b), c), and d) for x.

a) $\frac{1}{w}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ b) $2\sqrt{x-3}+\sqrt{3x-5}=8$

c) $\frac{x^2}{x^2-5x+6}=\frac{2}{x-2}+\frac{6}{(x-2)(x-3)}$ d) $\sqrt[4]{5x^2-6}=x$

2. Evaluate the following two expressions.

a) $\frac{(7.25)^{1359}\times\sqrt{(7.14)^{13.5}}}{(3.39)^{1481}}$ b) $\frac{(9.2)^{545}\times(5.33459)^{24.79}}{(4.15)^{934}\times\sqrt[7]{519.395}}$

3. A rectangular solid has the length of each side increased by the same amount in order to double the volume. Find the revised dimensions if the original dimensions are 3 by 4 by 5 cm.

4. A plane left an airfield to fly to a destination 1860 miles away. After flying at an unknown airspeed for 600 miles, the wind changed increasing the airspeed of the plane by 40 mph. This reduced the time of the trip by 45 minutes. What was the original airspeed of the plane?

5. A dealer bought a shipment of shoes for $480.00. He sold all but 5 pairs at a profit of $6.00 per pair, thereby making a total profit of $290.00 on the shipment. How many pairs of shoes were in the original shipment?

6. Two train stations A and B are 300 miles apart and in the same time zone. At 5AM a passenger train leaves A for B and a freight train leaves B for A. The two trains meet at a point 100 miles from B. Had the speed of the passenger train been 10 mph faster, it would have reached B 9 hours before the fright train reached A. How fast was each train traveling?

44. 2006 Survivalist Finance Exam *AR, AL, CL*

1. Find the final value of $250,000.00 deposited at 7% interest for a period of 10 years if **A**) the compounding is yearly, **B**) the compounding is quarterly, **C**) the compounding is monthly, and **D**) the compounding is continuous. **F**) Find the effective interest rate for each of the four cases **A**, **B**, **C**, & **D**.

2. A down payment of $5000.00 is made on a car costing $30,000.00.The rest is financed over a period of 6 years at 5% simple interest. **A**) What is the monthly payment? **B**) What is the total payback assuming the loan is kept to completion? **C**) How much of this payback is interest?

3. Fill in the table below comparing three possible mortgages where $230,000.00 is the amount borrowed.

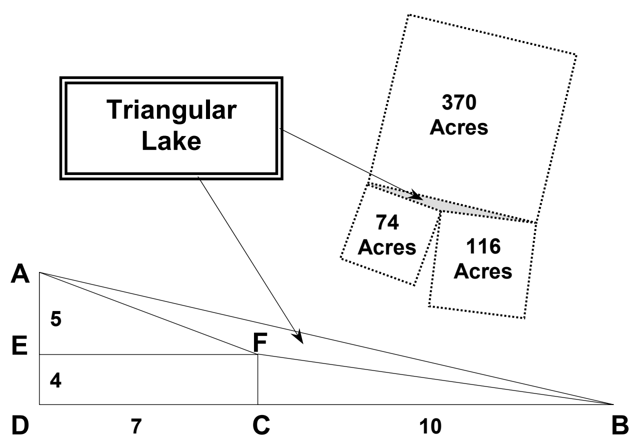
FIXED RATE MORTGAGE BORROWING $230,000.00				
Years	Interest Rate	Monthly Payment	Total Loan Payback	Total Interest Paid
30	6.50%			
20	6.25%			
15	5.50%			

4. You start an Individual Retirement Account (IRA) at age 25 by investing $7000.00 per year in a very aggressive growth fund having an annual rate of return that averages 13%. Five years later, you roll the proceeds into a blue-chip growth fund whose average long-term-rate-of-return is 9% annually. Concurrently, you increase your annual contribution to $9000.00 and continue this to age 69. **A**) Assuming continuous and steady interest rates, project the face value of your total investment when you reach age 69. **B**) What is the present value of the total projected in part **A**) if inflation holds at a steady rate of 3% throughout the 44-year period? **C**) Assuming the annuity pays a fixed 4% and is amortized at age 100, what is the present value of the monthly payment associated with an annuity bought at age 69 with the total in **B**). **D**) If you actually lived to be age 100, what would be the present value of the final annuity payment if the inflation rate remains relatively constant at 3% throughout this 75-year period?

45. Sam Lloyd's Triangular Lake *AL, GE*

Sam Lloyd was a famous American creator of puzzles, tricks and conundrums who produced most of his masterpieces in the late 1800s. Many of Lloyd's puzzles have survived and actually thrived, having found their way into modern puzzle collections. One of Sam Lloyd's famous creations is his Triangular Lake puzzle. Lloyd subtlety gives his readers two choices: solution by sweat and brute force, or solution by cleverness and minimal effort. The clever solution requires use of the Pythagorean Theorem. What follows is Lloyd's original statement:

Sam Lloyd's Challenge: "The question I ask our puzzlists is to determine how many acres there would be in that triangular lake surrounded (as shown in the figure below) by square plots of 370, 116 and 74 acres. The problem is of peculiar interest to those of a mathematical turn, in that it gives a positive and definite answer to a proposition, which, according to usual methods produces one of those ever-decreasing, but never-ending decimal fractions."



Triangle Lake with Hint

46. The Pythagorean Magic Square *AR, GE*

Magic squares of all types have intrigued math enthusiasts for decades. What you see below in is a Pythagorean masterpiece that couples three magic squares of different sizes (or *orders*) with the truth of the Pythagorean Theorem. Royal Vale Heath, a well-known British puzzle maker, created this wonder in England prior to 1930. Of the fifty numbers used in total, none appears more than once.

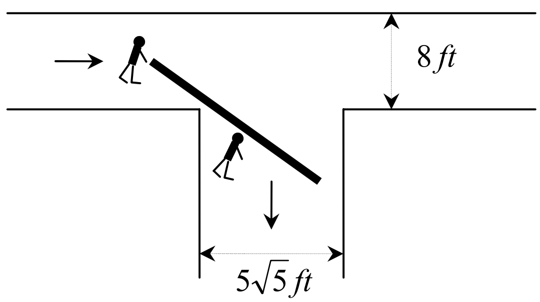
<table>
<tr><th colspan="12">THE PYTHAGOREAN 3-4-5 WONDER SET OF THREE MAGIC SQUARES</th></tr>
<tr><td colspan="3" rowspan="3">3X3 magic sum is 174. Square the sum of nine numbers to obtain 272,484</td><td colspan="4" rowspan="2">4X4 magic sum is also 174. Square the sum of sixteen numbers to obtain 484,416</td><td colspan="5">5X5 magic sum is also 174. Square the sum of twenty-five numbers to obtain 756,900</td></tr>
<tr><td>16</td><td>22</td><td>28</td><td>34</td><td>74</td></tr>
<tr><td>36</td><td>43</td><td>48</td><td>47</td><td>33</td><td>73</td><td>20</td><td>21</td><td>27</td></tr>
<tr><td>61</td><td>54</td><td>59</td><td>49</td><td>46</td><td>37</td><td>42</td><td>25</td><td>26</td><td>32</td><td>72</td><td>19</td></tr>
<tr><td>56</td><td>58</td><td>60</td><td>39</td><td>40</td><td>51</td><td>44</td><td>71</td><td>18</td><td>24</td><td>30</td><td>31</td></tr>
<tr><td>57</td><td>62</td><td>55</td><td>50</td><td>45</td><td>38</td><td>41</td><td>29</td><td>35</td><td>70</td><td>17</td><td>23</td></tr>
<tr><td colspan="12">$3^2 + 4^2 = 5^2$ **&** $272{,}484 + 484{,}416 = 756{,}900$ **!**</td></tr>
</table>

Pythagorean Magic Squares

Challenge: Check it out and explore!

47. The Famous Girder Problem *CL*

The problem below started to appear in calculus texts circa 1900. My father first experienced it in 1930 as an engineering student, and I first encountered it in the winter of 1966. It still appears in modern calculus textbooks disguised—and somewhat watered down—as a geometric optimization problem. The girder problem is famous because of the way it thoroughly integrates the principles of plane geometry, algebra, and differential calculus. My experience as a teacher has been that “many try, but few succeed. “ Will you? Have fun!



The problem and the challenge: Two people at a construction site are rolling steel beams down a corridor 8 feet wide into a second corridor $5\sqrt{5}$ feet wide and perpendicular to the first corridor. What is the length of the longest girder that can be rolled from the first corridor into the second corridor and continued on its journey in the construction site? Assume the beam is of negligible thickness.

48. One Mean Derivative *CL*

There is an old maxim of mathematics that says, "You really learn your algebra when you take calculus; and if you don't, calculus will take you." What follows is the first calculus problem in the book. But wait, is it more of an algebra problem. You decide!

Challenge: For the function $f(x)=\frac{1}{\sqrt{x^3}}$, find the derivative $f'(x)$ by appealing to the basic definition for $f'(x)$ which embodies the limit concept: $f'(x)=\lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$.

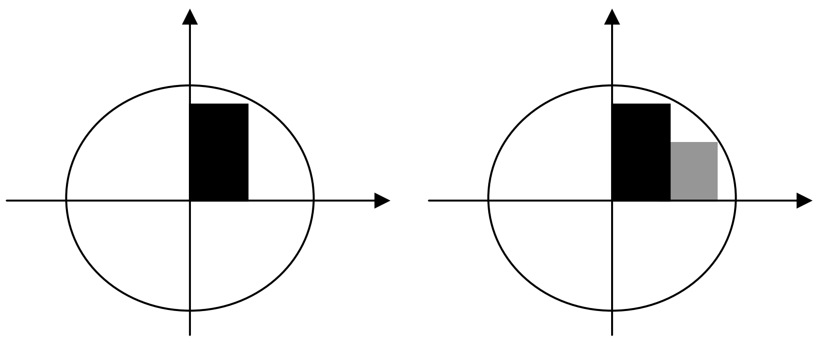
Additional Challenge: Brush off some dusty neurons and see if you can determine the equation of the line tangent to the graph of $f(x)$ at $x=4$. Also, determine the equation of the line normal to the graph of $f(x)$ at $x=4$.

49. No Calculators Allowed *CL*

Challenge: Use the techniques of differential calculus to prove the transcendental inequality $e^{\pi} > \pi^{e}$.

50. Double Rectangles *CL*

A frequent problem in first term calculus is to find the area of the largest rectangle that can be inscribed inside the first-quadrant portion of the unit circle. See the figure on the left below. For those of you who haven't worked with calculus for a while, I suggest that this well-known problem (which is solved using single-variable differential calculus) be your warm-up.



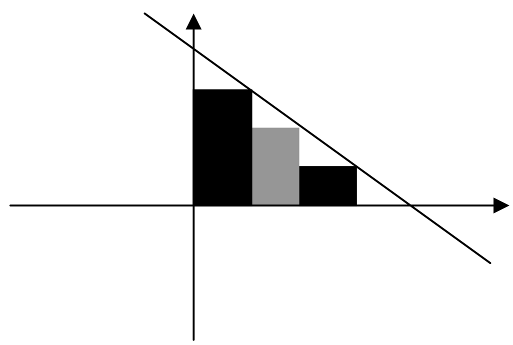
Challenge: It doesn't take much of an expansion to turn the above problem into a connoisseur's absolute delight consisting of two independent variables, partial derivatives, and subtle observations—not to mention a king-sized scoop of algebra. Here it comes! The figure on the right is joined to the following question: Find the dimensions for each of the two rectangles inscribed in the unit circle as shown that maximizes the total combined area of both rectangles.

Note: An Air Force Captain first introduced me to this problem in 1981 after experiencing it on a Ph.D. qualifying examination the week before. Thanks Joe for a superb treat!

51. Triple Play *CL*

Calculus is always more fun when two or three variables are in the game. The problem below is similar to the one presented in “A Mathematician’s Desert”, but we have added one more variable to the lineup!

Challenge: Consider the figure below where the equation of the line segment is given by $y = 1 - x$ where $0 \le x \le 1$.



Find the dimensions for each of the three rectangles inscribed in the triangular region as shown that maximizes the total combined area of the threesome.

$$\int_a^b f(x)dx$$

52. Calling all Data Lovers *ST*

There is a small, well-kept cemetery close to where I live, which is the final resting-place for 180 (last count) Catholic priests and brothers. Below is the data summary from all 180 headstones. Each four-digit entry is the year of death (with the 19 omitted) and age at death. For example, the first entry 6245 codes a death in 1962 at age 45.

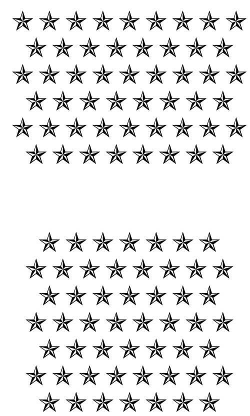
6245, 6286, 6338, 6346, 6383, 6393, 6462, 6464, 6475, 6488, 6557, 6671, 6679, 6682, 6763, 6784, 6832, 6839, 6846, 6854, 6866, 6876, 6877, 6877, 6883, 6884, 6952, 6957, 6984, 7033, 7059, 7065, 7072, 7079, 7086, 7087, 7143, 7167, 7168, 7176, 7182, 7189, 7236, 7252, 7261, 7275, 7287, 7288, 7356, 7369, 7462, 7467, 7468, 7471, 7474, 7478, 7550, 7666, 7667, 7667, 7676, 7678, 7682, 7690, 7741, 7764, 7774, 7775, 7784, 7967, 7968, 7969, 7972, 7974, 7977, 7990, 8082, 8084, 8164, 8167, 8170, 8172, 8182, 8182, 8183, 8184, 8191, 8246, 8259, 8266, 8275, 8276, 8286, 8290, 8294, 8373, 8376, 8378, 8385, 8468, 8474, 8477, 8479, 8480, 8567, 8569, 8569, 8569, 8570, 8579, 8580, 8584, 8666, 8672, 8673, 8678, 8681, 8769, 8769, 8774, 8781, 8790, 8864, 8870, 8878, 8888, 8889, 8954, 8973, 8973, 8979, 8990, 8993, 9067, 9082, 9083, 9088, 9149, 9155, 9171, 9176, 9181, 9183, 9260, 9276, 9294, 9357, 9368, 9380, 9380, 9385, 9390, 9392, 9467, 9471, 9483, 9483, 9484, 9486, 9486, 9568, 9580, 9583, 9583, 9584, 9658, 9662, 9677, 9677, 9678, 9679, 9680, 9682, 9684, 9686, 9689, 9693, 9781, 9788, 9797: **R.I.P.**

Challenge: can you argue a case for increasing male longevity in the United States from this particular sample? Why or why not? Let statistics (or sadistics as some of my students like to say) be your guide.

Answers to Selected Puzzles

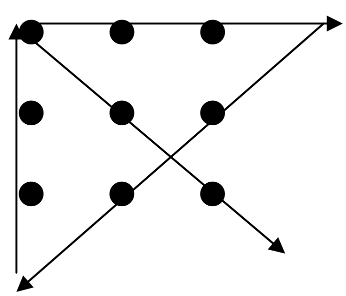
By page and problem number **(Page #, Problem #)**

(8, 1)



(10, 3)

First:



Second: 30 squares total

(13, 6): 31 in the frame

(13, 7): Step 5, division by 0

(14, 8, A): F, W, G, C: F, G cross; F comes back; F, W cross; F,G come back; F, C cross; F comes back; F, G cross for the last time.

(14, 8, B): E, B cross (2 minutes); E comes back (2 minute); A, L cross (10 minutes), B comes back (1 minute), E, B cross for the second time (2 minutes). Times total 17 minutes.

(18, 12): Yes. The magic constant is 45.

(19, 13): Unlike the magic square, the solution below is only one of many.

2	4	7
5	1	8
9	3	6

(21, 15): The magic sum for the nested 3 by 3 is 29.

(27, 22): Do, or do we not have triangles!

(32, 28): 84 years old

(32, 29): Gauss added symmetric pairs from the two extreme ends of the series, $1+2+3+....+98+99+100$, starting outward and working his way in towards the center. Each pair summed to 101. There are fifty such pairs. Hence the total is 5050. The answer to $1+2+...+2005+2006$ is $1003 \cdot 2007 = 2{,}013{,}021$.

(33, 30):

Stage	3 Men	Hotel	Bell	Sum
1) Before entering	30	0	0	30
2) Front Desk	0	30	0	30
3) Send Back	0	25	5	30
4) Distribution	3	25	2	30

(35, 34): 533 1/3 bananas

(39, 38):

G	W	B	Y
B	Y	R	G
R	G	W	B
W	B	Y	R

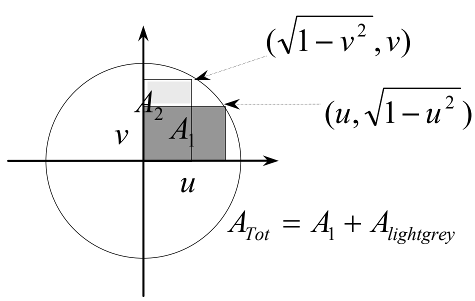
(47, 45): 11 Acres

(49, 47): Exactly 27 feet

(50, 49) : Hint, use derivatives to analyze the function

$f(x) = e^x - x^e$ on the interval $0 \le x \le \infty$.

(51, 50): See the diagram below. Note the definitions of u and v.



Setup for Double Rectangles

Next page

$$A_{Tot}(u,v) = u\sqrt{1-u^2} + (v - \sqrt{1-u^2})\sqrt{1-v^2}$$

implies

$$\frac{\partial A}{\partial u} = \frac{1 + u\sqrt{1-v^2} - 2u^2}{\sqrt{1-u^2}} = 0$$

$$\frac{\partial A}{\partial v} = \frac{1 + v\sqrt{1-u^2} - 2v^2}{\sqrt{1-v^2}} = 0$$

implies

$$u = v \Rightarrow$$

$$u = 0.8507 \,\&\, 0.5257 \Rightarrow$$

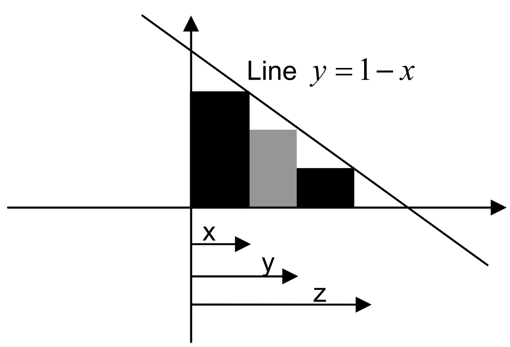
$$A_{Tot}(0.8507, 0.8507) = 0.618$$

The reader is left to check out the solution $u = 0.5257$.

(52, 51): Set up the area function

$$A(x, y, z) = x(1-x) + (y-x)(1-y) + (z-y)(1-z)$$

and optimize for the region $0 \le x \le y \le z \le 1$.



(53, 52): The graph below is just one example of what is possible.